$$\hat{x}(k) = M_1(k)\hat{x}_1^*(k) + M_2(k)\hat{x}_2^*(k) \tag{26}$$

where $\hat{x}_1^*(k)$, $\hat{x}_2^*(k)$ are generated by

$$\hat{x}_1^*(k) = F_1^*(k-1)\hat{x}_1^*(k-1) + K_1^*(k)(y(k) - H_1^*(k)F_1^*(k-1)\hat{x}_1^*(k-1)) \tag{27}$$

$$K_1^*(k) = S_1^*(k)(H_1^*(k))^T(H_1^*(k)S_1^*(k)(H_1^*(k))^T + R(k))^{-1} \tag{28}$$

$$S_1^*(k) = F_1^*(k-1)P_1^*(k-1)(F_1^*(k-1))^T + G_1^*(k-1)Q(k-1)((G_1^*(k-1))^T \tag{29}$$

$$P_1^*(k) = (I - K_1^*(k)H_1^*(k))S_1^*(k) \tag{30}$$

and

$$\hat{x}_2^*(k) = F_{21}^*(k-1)\hat{x}_1^*(k-1) + F_2^*(k-1)\hat{x}_2^*(k-1) + K_2^*(k)(y(k) - H_1^*(k)F_1^*(k-1)\hat{x}_1^*(k-1)) \tag{31}$$

$$K_2^*(k) = S_{21}^*(k)(H_1^*(k))^T(H_1^*(k)S_1^*(H_2^*(k))^T + R(k))^{-1} \tag{32}$$

$$S_{21}^*(k) = F_2^*(k-1)P_{21}^*(k-1)(F_1^*(k-1))^T + F_{21}^*(k-1)P_1^*(k-1)(F_1^*(k-1))^T + G_2^*(k-1)Q(k-1)(G_1^*(k-1))^T \tag{33}$$

$$P_{21}^*(k) = S_{21}^*(k) - K_2^*(k)H_1^*(k)S_1^*(k) \tag{34}$$

The initial conditions are $\hat{x}_1^*(0) = N_1(0)\hat{x}(0)$, $\hat{x}_2^*(0) = N_2(0)\hat{x}(0)$, $P_1^*(0) = N_1(0)P(0)N_1^T(0)$, and $P_{21}^*(0) = N_2(0)P(0)N_1^T(0)$. The matrices at time k are defined by $K_1^* = N_1K$, $K_2^* = N_2K$, $S_1^* = N_1SN_1^T$, $S_{21}^* = N_2SN_1^T$, $P_1^* = N_1PN_1^T$, and $P_{21}^* = N_2PN_1^T$.

Notice that the order of matrix Riccati equation is reduced from $n \times n$ [Eqs. (24-25)] to $l \times l$ matrices S_1^* and P_1^* and $(n-l) \times l$ matrices S_{21}^* and P_{21}^* are involved in filter gain computation instead of full $n \times n$ matrices S and P. We note that Eqs. (27-30) constitute a self-governing, l-state Kalman filter of observable states $x_1^*(k)$, while Eqs. (31-34) form $(n-l)$ state filter of unobservable states $x_2^*(k)$ for given $\hat{x}_1^*(k-1)$, $P_1^*(k-1)$, and $S_1^*(k)$.

We also point out that unobservable states become "observable" over a sufficiently long time period if these unobservable states are asymptotically stable and coupled to observable states.

Lemma 4 (asymptotic observability): Let F_{21}^* be of full rank and

$$\psi(i+j,i) = F_2^*(i+j-1)F_2^*(i+j-2)\ldots F_2^*(i)$$

If $\|\psi(k,0)\|$ is bounded and goes to zero as $k \to \infty$, then unobservable states x_2^* are asymptotically observable.

Proof: We look at Eq. (21) with $w(k) = 0$. F_{21}^* couples x_2^* to x_1^*. The above condition on $\psi(k,0)$ makes an unforced system $x_2^*(k+1) = F_2^*(k)x_2^*(k)$ asymptotically stable.[8] By

$$x_2^*(k) = \psi(k,0)x_2^*(0) + \sum_{i=1}^{k} \psi(k,i)F_{21}^*(i-1)x_1^*(i-1)$$

the initial uncertainty $x_2^*(0)$ vanishes asymptotically, and x_2^* approaches asymptotically to the observable term of $\{x_1^*\}$. The extension to a stochastic case is straightforward by lemma 1. QED.

The significance of lemma 4 stems from the fact that in applications unknown constant biases (or parameters) are modeled as exponentially correlated random variables, which are indeed asymptotically observable, and incorporated into Kalman filtering where our order reduction by theorem 1 will be found valuable for reducing the computational load.

IV. Conclusion

We made clear that 1) when some states are not observable, the matrix Riccati equation of Kalman filter is order-reducible; and 2) if they are coupled to observable states and symptotically stable, then the unobservable states become "asymptotically" observable in the filter. The extension to the continuous-time and extended Kalman filters[7] is simple.

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Satellite Attitude Estimation from Communications Signal Strength

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Nomenclature

a = attitude unit vector
b = antenna boresight pointing unit vector
b_0 = null pointing direction, constrained to lie in (L_p, a) plane
H = matrix of partials of S_c with respect to $\hat{X} = \left[\frac{\partial S_c}{\partial x_1} \frac{\partial S_c}{\partial x_2}\right]$
L_p = vector from pilot station to spacecraft $= r - r_p$
r = vector from earth center to spacecraft

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r_g = vector from earth center to communications ground station
r_p = vector from earth center to pilot station
S = measured signal strength
S_c = computed signal strength
X = attitude state vector $[x_1, x_2]^T$, consisting of the two equatorial plane components of a
$\hat{X}$ = attitude state estimate
β = angle between spin axis and boresight
ν = angle in spin plane from b_0 to boresight
ρ = unit vector from spacecraft to communications ground station = $(r_g - r)/|r_g - r|$

Introduction

FOLLOWING the temporary failure of one of two telemetry transmitters on the Indonesian communications satellite Palapa-2, it was decided to investigate the problem of controlling this class of spacecraft in the absence of telemetered data. An important part of this control function is the determination of spin axis attitude, which in this case must be maintained within 0.25 deg of orbit normal attitude for satisfactory communications. Normally, this determination is made to an accuracy of 0.025 deg from infrared earth sensor and sun sensor data telemetered to a ground station.[1] With only one onboard telemetry transmitter operating, the system was left without a conventional backup source of attitude data. This communications satellite however, has a shaped antenna pattern so that signal strength is sensitive to attitude change, especially near the edges of the pattern. This sensitivity is, of course, the reason the attitude is controlled in the first place. It is natural to exploit this connection between attitude and signal strength, which has the attractive feature that attitude deviations large enough to adversely affect communications signal strength are inherently observable while those small enough not to noticeably affect communications are of no consequence.

The Master Control Station (MCS) for the Indonesian system is located near Jakarta, which is a point of high North-South gain slope in the communications antenna pattern, making it the obvious site for data collection. Automatic signal strength measuring equipment was therefore installed at this site. The signal strength is expected to show a nearly sinusoidal variation over the course of one day (the spacecraft orbital period), but corrupted by noise introduced principally by rain attenuation and ionospheric scintillation. It is difficult to predict and measure the absolute signal strength accurately, so it was decided to use the variation in signal strength from an arbitrary reference as the data type. Isolating the diurnal component of variability and comparing it against predicted variations in a least-squares filter constituted the basic approach to attitude estimation. The signal strength variation as recorded by the instrumentation is itself a function of signal strength, so it was necessary to calibrate the measuring system. This was done by repeating every measurement with a 1-dB attenuator switched into the system. A series of such measurements allowed the relationship between signal strength variation (in dB) and signal strength (in V) to be established empirically. The measurements were made every 10 minutes, allowing the equipment to have a long enough time constant to smooth rapid fluctuations.

Control System and Signal Strength Model

The dual-spin spacecraft has an antenna mounted with its boresight at a fixed angle β from the spin axis a, as shown in Fig. 1. It is, however, free to rotate about the spin axis, and is held at the correct azimuth (despun) by a pilot signal from a ground station. There is a null in the antenna pattern, obtained by appropriate differencing of feed horn signals, that is used by the antenna pointing control system to maintain the boresight pointed at an azimuthal angle ν from the azimuth of the pilot source. The elevation of this source in spacecraft coordinates is normally variable over a day due to orbital motion. The angle ν is a known function of elevation (known from antenna calibration), although the variation is so small under normal operating conditions as to be ignorable for the purposes of this work.

The antenna pointing model assumes that the boresight is maintained at a known angle ν from the pilot tone direction. Given the attitude, or an initial guess, it is possible to compute the azimuth and elevation in spacecraft coordinates of the line of sight to the MCS where signal strength is being measured. Antenna performance is calibrated before launch in terms of gain as a function of azimuth and elevation. The gain is therefore obtained by two-dimensional interpolation once the azimuth and elevation have been computed. Both uplink and downlink patterns are stored in this manner, although the former is not used if the uplink carrier has driven the traveling wave tube into saturation.

At a given time, the boresight direction is

$$b = \sin\beta \cos\nu a_n + \sin\beta \sin\nu b_n + \cos\beta a \tag{1}$$

where

$$b_n = (L_p \times a)/|L_p \times a| \qquad a_n = b_n \times a$$

The azimuth and elevation of the vector ρ relative to the antenna boresight is required. The coordinate system (x,y,z) based on vectors a and b is defined by $y = (b \times a)/|b \times a|$, $x = b \times y$, $z = b$. In this system the elevation E and azimuth A of ρ are

$$E = \sin^{-1}(\rho \cdot x) \tag{2}$$

$$A = \tan^{-1}([\rho \cdot y]/[\rho \cdot z]) \tag{3}$$

Given these angles, tables of up- and downlink gain are entered, and the expected signal strength S_c is found by two-dimensional interpolation. The model described here to predict the signal strength was used in experiments with the Anik-A spacecraft of Telesat Canada.[2] It was found that measured and predicted azimuths agreed well, the difference between them showing an approximately sinusoidal variation over an orbital period with peak-to-peak amplitude of 0.1 deg. This was interpreted as a diurnal oscillation in ν.

Data Processing

Signal strengths are measured in volts with an approximately 1-dB attenuator alternately switched out and in the measuring circuit. The sensitivity of signal strength variation to change in measured voltage varies regularly with voltage, so it was decided to represent the functional dependence with a quadratic expression whose coefficients are obtained from a least-square curve-fitting routine. Data points more than a specified number of standard deviations from the fitted curve are rejected. This process is repeated as

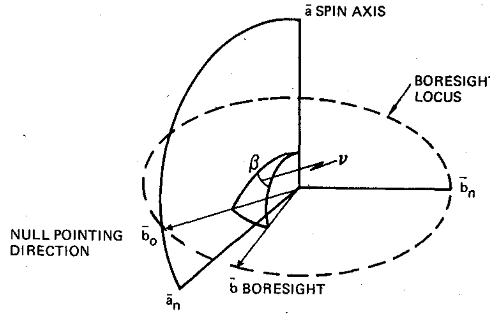

Fig. 1 The antenna boresight b is mechanically constrained to an angle β from the spacecraft spin axis a, and is then controlled to point at an angle ν from the null pointing direction b_0.

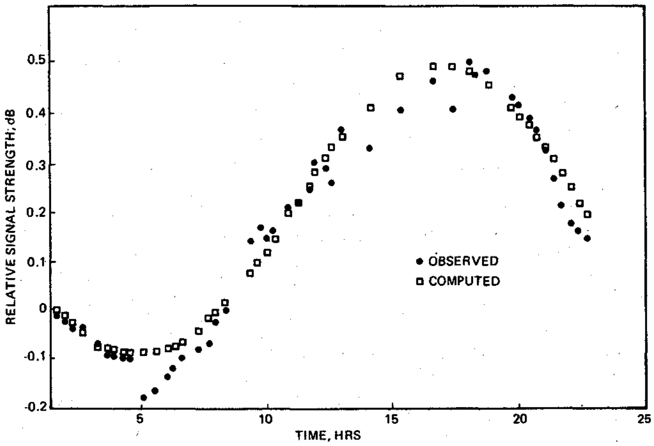

Fig. 2 Observed and computed signal strength variations relative to the first data point for a one-day period.

many times as necessary. The signal strength S is obtained by integrating this quadratic expression.

This data is expected to vary approximately sinusoidally with the spacecraft orbital period, and is therefore curve-fitted to such a function. Once again points more than a specified number of standard deviations from the sine curve are removed, eliminating the noisiest data. The process is helped by selecting stretches of data that show this expected variation. Generally, these stretches are associated with periods of clear dry weather. The data used in the estimation routine is not the absolute signal strength but rather the variation ΔS relative to the first data point. That point is taken from the sinusoidal expression, rather than being the initial measurement itself. A plot of measured ΔS vs time is given in Fig. 2.

The expected signal strength variation ΔS_c corresponding to each of the N measured values not rejected during preprocessing is used to form the residuals required to differentially correct an initial guess of the attitude state vector:

$$\hat{X}_{k+1} = \hat{X}_k + \left(\sum_{i=1}^{N} H_i^T H_i\right)^{-1} \sum_{i=1}^{N} H_i^T(\Delta S_i - \Delta S_{ci}) \tag{4}$$

In these computations the matrix H_i is obtained numerically and again, since the measurements are relative to the first data point, the H matrices are modified by $H_i - H_i - H_1$. In addition, the attitude is adjusted for solar torque precession based on a model which has been shown to predict quite well the long-term attitude motion.[3]

The computed signal variation after six differential corrections is included in Fig. 2. The corresponding differences between observed and computed values, or residuals, are less than 0.1 dB. Ideally, the residuals should be randomly distributed with zero mean, a condition which these results come close to meeting. Tests of this attitude determination scheme were made with data taken in Jakarta during 1978. It was found that the attitude estimates agreed well with those determined using sun and earth sensor data. In all cases the difference was a small fraction of the allowed control band of ±0.25 deg. Thus the somewhat simplified model of the antenna control system, which assumes essentially perfect control, and the lack of modelling for environmental effects such as temperature, atmospheric moisture and ionospheric variations, appear justified.

Conclusions

Although the Palapa-2 communications satellite still has an operating telemetry system, it has nevertheless been demonstrated that the spacecraft attitude can be measured without it. Comparison with standard measurement techniques (data from infrared earth sensors and a sun sensor) shows that the attitude can be determined to well within the control requirement for satisfactory communications. This has been accomplished by measurement at a single earth station. Greater accuracy can be obtained by adding data from a station in the northern part of the antenna pattern such as Manila, where the signal strength variation is out of phase with that experienced in Jakarta.

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Technical Comments

Comments on "Millimeter-Wave Missile Seeker Aimpoint Wander Phenomenon"

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THE subject paper[1] provides interesting experimental results from a "millimeter-wave" missile seeker. However, considerable speculative and misleading discussion is provided to explain a phenomenon that has been extensively covered in the literature. The authors state: "Indeed, no study of the aimpoint wander phenomenon itself has been found in the literature by the authors." However, the authors also use the term "glint" which is a whole category in the International Cumulative Index on Radar System.[2] The glint category has many papers on the general subject of "aimpoint wander" which is called target noise in the IEEE Standard Dictionary of Electrical and Electronic Terms.[3] The subject of "aimpoint wander" is also included in the Radar Handbook[4] as a chapter entitled "Target Noise." The target noise is composed of two sources of angle errors: the amplitude noise (source affecting only conical scan radars as discussed in the subject paper), and angle noise (not recognized by the authors)—a noise-like variation in the apparent angle of arrival of the echo received from a target. Angle noise was first recognized in the early 1950's when it was found that

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